Engr210a Lecture 19: Summary and conclusions

- Structured singular value computation
- Arbitrary uncertainty
- LTI uncertainty
- Further topics

The matrix structured singular value

$$\mu(M, \mathbf{\Delta}) = \frac{1}{\inf\Big\{ \|\Delta\| \;;\; \Delta \in \mathbf{C\Delta},\; I - M\Delta \text{ is singular} \Big\}}$$

$$\mathbf{C\Delta}_{\mathsf{s,f}} = \Big\{ \Delta = \operatorname{diag}\big(\delta_1 I_{m_1}, \ldots, \delta_s I_{m_s}, \Delta_{s+1}, \ldots, \Delta_{s+f}\big) \;;\; \delta_i \in \mathbb{C},\; \Delta_k \in \mathbb{C}^{m_k \times m_k} \Big\}$$

Theorem

$$\mu(M, \mathbf{\Delta}_{\mathsf{s,f}}) \leq \inf\Big\{ \|\Theta M \Theta^{-1}\| \;;\; \Theta \in \mathbf{\Theta} \Big\}$$

Notes

- If $2s + f \leq 3$, then $\mu(M, \mathbf{\Delta}_{\mathsf{s,f}}) = \inf\Big\{ \|\Theta M \Theta^{-1}\| \;;\; \Theta \in \mathbf{\Theta} \Big\}$
- In general, computing μ is NP-hard.

Arbitrary uncertainty

Let

$$\mathbf{C\Delta_a} = \Big\{ \Delta = \operatorname{diag}(\Delta_1, \ldots, \Delta_d) \ ; \ \Delta \in \mathcal{L}(L_2) \Big\}$$

Given $M \in RH_\infty$, we have

$$\mu(M, \mathbf{\Delta_a}) = \inf \Big\{ \|\Theta M \Theta^{-1}\| \ ; \ \Theta \in \mathbf{\Theta} \Big\}$$

Notes

- For arbitrary operator uncertainty, this gives a necessary and sufficient condition for robust well-connectedness.
- But this uncertainty class includes arbitrarily time-varying operators.

LTI uncertainty

- For LTI uncertainty, we have again

$$\mu(M, \mathbf{\Delta_{TI}}) \le \inf \Big\{ \|\Theta M \Theta^{-1}\| \ ; \ \Theta \in \mathbf{\Theta_{TI}} \Big\}$$

Further topics

- μ synthesis, and the DK iteration.
- Nonlinearities, IQCs, and quadratic stability.
- Discrete and hybrid systems.
- The gap metric.
- Partial differential equations.
- Spatially distributed systems.
- Multi-objective control.